

Cerenkov's Effect and Neutrino Oscillations in Loop Quantum Gravity

G. Lambiase^{a,b,*}

^a*Dipartimento di Fisica "E.R. Caianiello" Università di Salerno, 84081 Baronissi (Sa), Italy.*

^b*INFN - Gruppo Collegato di Salerno, Italy.*

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Bounds on the scale parameter \mathcal{L} arising in loop quantum gravity theory are derived in the framework of Cerenkov's effect and neutrino oscillations. Assuming that \mathcal{L} is an universal constant, we infer $\mathcal{L} \gtrsim 10^{-18} \text{eV}^{-1}$, a bound compatible with ones inferred in different physical context.

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I. INTRODUCTION

The difficulties to build up a complete theory of quantum gravity has motivated the development of semiclassical approaches in which a Lorentz invariance breakdown occurs at the effective theory level. The deformations of the Lorentz invariance manifest by means of a slight deviation from the standard dispersion relations of particles propagating in the vacuum. Such modifications has been proposed in the paper [1], and derived in two different approaches: String Theory [2] and Loop Quantum Gravity [3–5].

In this work we shall refer to the Alfaro, Morales-Técotl, Urrutia (AMU) paper [4,5] which is based on the loop quantum gravity [6]. In this model a new length scale \mathcal{L} occurs, with $\mathcal{L} \gg l_{Pl} \sim 10^{-19} \text{GeV}^{-1}$. The scale \mathcal{L} separates the distances d that manifest the quantum loop structure of space ($d \ll \mathcal{L}$) from the continuous flat space ($d \gtrsim \mathcal{L}$). As observed in [7], this length scale might give rise to some observable effect of quantum gravity.

In the AMU formalism, the dispersion relation of freely propagating fermions and photons have the form

$$E_i^2 = \tilde{A}_i^2 p_i^2 + m_i^2, \quad (\text{I.1})$$

where \tilde{A}_i encodes the Lorentz invariance deformation \mathcal{L} -dependent and is different for different species of particles. More specifically, for Majorana fermions [4], the dispersion relation is given by

$$E_{\pm}^2 = A_p^2 p^2 + \eta p^4 \pm 2\lambda p + m^2, \quad (\text{I.2})$$

where

$$A_p = 1 + \kappa_1 \left(\frac{l_{Pl}}{\mathcal{L}} \right) + \kappa_2 \left(\frac{l_{Pl}}{\mathcal{L}} \right)^2, \quad \eta = \kappa_3 l_{Pl}^2, \quad \lambda = \kappa_5 \frac{l_{Pl}}{2\mathcal{L}^2}. \quad (\text{I.3})$$

The \pm signs stand for the helicity of the propagating fermions, and κ_i are unknown coefficients of the order $\kappa_i \sim \mathcal{O}(1)$.

For photons, the dispersion relation derived in AMU approach is [5]

$$\omega_{\pm}^2 = k^2 [A_{\gamma}^2 \pm 2\theta_{\gamma} l_{Pl} k], \quad (\text{I.4})$$

where

$$A_{\gamma} = 1 + \kappa_{\gamma} \left(\frac{l_{Pl}}{\mathcal{L}} \right)^{2+2Y}. \quad (\text{I.5})$$

Again \pm signs stand for the helicity dependence of photons in the dispersion relation, $\kappa_{\gamma} \sim \mathcal{O}(1)$, $\theta_{\gamma} \sim \mathcal{O}(1)$, and $Y = -1/2, 0, 1/2, 1, \dots$. It is worth note that a similar result has been obtained by Gambini and Pullin [3] with $A_{\gamma} = 1$ and Ellis et al. [2] with the difference that the helicity dependence is absent.

The aim of this paper is to determine bounds on the scale length \mathcal{L} . Alfaro and Palma [7] applies the AMU theory to the observed Greisen-Zatsepin-Kuz'min (GZK) limit anomaly and to the so called TeV- γ paradox, i.e. the detection of high-energy photons with a spectrum ranging up to 24 TeV from Mk 501, a BL Lac object at a red-shift of 0.034

*E-mail: lambiase@sa.infn.it

($\sim 134\text{Mpc}$). Assuming that no anomalies there exist, as recent works seem to indicate [8], they find that, with a specific choice of the κ_i -parameters, the favorite range for \mathcal{L} is ($Y = 0$)

$$4.6 \times 10^{-17} \text{eV}^{-1} \gtrsim \mathcal{L} \gtrsim 8.3 \times 10^{-18} \text{eV}^{-1}. \quad (\text{I.6})$$

Bounds on \mathcal{L} are inferred in two different contexts: *i*) the emission of radiation by charged particles via Cerenkov effect; *ii*) neutrino oscillations. The analysis is carried out for \mathcal{L} universal constant, and $k_3 = 0$ and $k_5 = 0$ in (I.1). Besides, to explore the leading order helicity independent effects of photons, we also put $\theta_\gamma = 0$. A detailed study including the helicity term in (I.4) has been recently performed by Jacobson et al. in [9].

II. CERENKOV EFFECT IN THE AMU THEORY

This Section is devoted to study the emission of the Cerenkov radiation by a charged particle whose dispersion relation is modified by the microscopic loop structure of space. As well known, the Cerenkov effect causes the charged particles to decelerate radiatively, and the spectrum of the radiated energy might be used as a sensitive means for probing their properties. This effect has been also studied in [9,10], but the method to derive bounds on \mathcal{L} is different.

The dispersion relation of photons (I.4) allows to infer the phase velocity v_γ

$$v_\gamma \equiv \frac{\omega}{k} \sim 1 + (A_\gamma - 1). \quad (\text{II.1})$$

For massive particles, the groups velocity v_p , corresponding to the physical velocity of the particles, is obtained by deriving the energy (I.2) with respect to the momentum p

$$v_p \equiv \frac{dE}{dp} \sim 1 + (A_p - 1) - \frac{m^2}{2p^2}. \quad (\text{II.2})$$

The ratio between the phase velocity (II.1) and the group velocity (II.2) is ($p \simeq E$)

$$\frac{v_\gamma}{v_p} = 1 + \frac{m^2}{2p^2} - (A_p - A_\gamma) - (A_p - 1)(A_\gamma - 1). \quad (\text{II.3})$$

The Cerenkov process occurs if the condition $v_\gamma/v_p < 1$ holds, i.e.

$$\frac{m^2}{2p^2} - (1 + A_p A_\gamma - 2A_\gamma) < 0. \quad (\text{II.4})$$

In the limit $\mathcal{L} \rightarrow \infty$ the above relation reduces to $m^2 < 0$, so that the Cerenkov effect is not kinematically allowed.

From Eq. (II.4) one defines the threshold energy E_0

$$E_0 = \frac{m}{\sqrt{2[1 + A_\gamma(A_p - 2)]}}, \quad (\text{II.5})$$

above which a charged particle is faster than the corresponding phase velocity of light and must then emit Cerenkov radiation.

The rate of the emitted energy by the charged particle is [11]

$$\begin{aligned} W(E) &= \frac{dE}{dt} = e^2 \int_0^{\bar{\omega}} \left(1 - \frac{v_\gamma^2}{v_p^2} \right) \omega d\omega \\ &= e^2 m^2 \frac{(E^2 - E_0^2)(E - E_0)^2}{2E_0^2 E^2}, \end{aligned} \quad (\text{II.6})$$

where the integration is overall the possible emitted frequencies $\bar{\omega} = E - E_0$ and Eqs. (II.3) and (II.5) have been used. The distance that charged particles can travel before to relax their energy as Cerenkov radiation is

$$\begin{aligned} L(E_f) &= \int_{x_f}^{x_i} dx = - \int_{E_i}^{E_f} \frac{dE}{W(E)} \\ &\simeq \frac{E_0}{4e^2 m^2} \left\{ \ln \frac{(E_i - E_0)(E_f + E_0)}{(E_i + E_0)(E_f - E_0)} + 6E_0 \left(\frac{1}{E_f - E_0} - \frac{1}{E_i - E_0} \right) + 2E_0^2 \left[\frac{1}{(E_f - E_0)^2} - \frac{1}{(E_i - E_0)^2} \right] \right\} \end{aligned} \quad (\text{II.7})$$

where E_i denotes the initial energy of the particle, and E_f is the final energy which differs from the threshold energy E_0 by an infinitesimal quantity, $E_f = (1 + \zeta)E_0$, with $\zeta \ll 1$ ($E_i > E_f > E_0$). In the limit $E_i \gg E_0$, Eq. (II.7) reduces to $L(E_f) \sim E_0/2e^2m^2\zeta^2$. Thus the loop quantum gravity effects become important on distance of cosmological relevance, provided ζ is extremely small. In fact, for $L(E_f) \sim \text{Mpc}$ and taking $E_0 \sim 10^9 \text{GeV}$ as threshold energy, it follows $\zeta < 10^{-8}$. Notice that $E_f = E_0$ corresponds to the asymptotic value $L(E_f) \rightarrow \infty$.

To determine a bound on the parameter \mathcal{L} , we shall work in the context of cosmic rays physics. As widely believed, cosmic rays have extragalactic origin [12–14] with energies exceeding 10^9GeV , and sources located at distances greater than $\sim \text{few Mpc}$. Besides, at the energy greater than 10^{10}GeV , the spectrum should contain mostly protons [13,15]. The large number of observed events with energies exceeding 10^{10}GeV suggests that Cerenkov effect occurs with a threshold energy expected to be of the order $E_0 \gtrsim 10^9 \text{GeV}$. For protons with $m \sim \text{GeV}$ and for $Y = -1/2, 0$, we get

- $Y = -1/2$. In such a case $A_\gamma \sim 1 + \kappa_\gamma(l_{Pl}/\mathcal{L})$, and we put $\kappa_2 = 0$ in the expression for $A_p \sim 1 + \kappa_1(l_{Pl}/\mathcal{L})$. Being $\delta\kappa_1 \equiv \kappa_1 - \kappa_\gamma \sim \mathcal{O}(1)$ ($\kappa_1 > \kappa_\gamma$), it follows

$$\mathcal{L} = \frac{2\delta\kappa_1 l_{Pl} E_0^2}{m^2} \gtrsim 10^{-8} \text{eV}^{-1}. \quad (\text{II.8})$$

This bound is nearly in the range of nuclear physics, so that it can be discarded since no evidence of loop structure occurred at this scale.

- $Y = 0$. The coefficient A_γ reduces to $A_\gamma \sim 1 + \kappa_\gamma(l_{Pl}/\mathcal{L})^2$, and we put $\kappa_1 = 0$ thus $A_p \sim 1 + \kappa_2(l_{Pl}/\mathcal{L})^2$. Being $\delta\kappa_2 \equiv \kappa_2 - \kappa_\gamma \sim \mathcal{O}(1)$ ($\kappa_2 > \kappa_\gamma$), it follows

$$\mathcal{L} = \frac{\sqrt{2\delta\kappa_2} l_{Pl} E_0}{m} \gtrsim 10^{-18} \text{eV}^{-1}. \quad (\text{II.9})$$

This bound agrees with (I.6) derived in Ref. [7].

III. NEUTRINO OSCILLATIONS IN AMU THEORY

The possibility that neutrinos particles may oscillate in different flavor states is the most discussed problem of today theoretical and experimental physics. Neutrino oscillations in the vacuum occur owing to the non-degeneration of the mass-matrix eigenvalues and to the difference of the mass eigenstates $|\nu_1\rangle$, $|\nu_2\rangle$ from the weak interaction eigenstates $|\nu_e\rangle$, $|\nu_\mu\rangle$. In the standard scenario, the energy splitting between two mass eigenvalues $E_{iM} = m_i^2/2p$, $i = 1, 2$, is $\Delta E = \Delta m^2/2p$, where $\Delta m^2 = m_1^2 - m_2^2$ is the mass squared difference. The oscillation length is given by¹ $L_M = 2\pi/\Delta E$.

In AMU's theory, the energy of particles is modified according to Eq. (I.2):

$$E_i = p + (A_p^i - 1)p + \frac{m_i^2}{2p}, \quad i = 1, 2, \quad (\text{III.1})$$

where

$$A_p^i = 1 + \kappa_1^{(i)} \frac{l_{Pl}}{\mathcal{L}} + \kappa_2^{(i)} \left(\frac{l_{Pl}}{\mathcal{L}} \right)^2. \quad (\text{III.2})$$

The parameters κ_j , $j = 1, 2, \dots$ are different for all particles. It is hence natural to assume that κ_1 and κ_2 are flavor depending. In this new setting, the energy splitting is

$$\Delta E \approx \frac{\Delta m^2}{2p} + \delta A_p p, \quad (\text{III.3})$$

¹It is worth note that the dispersion relation can be also modified by taking into account the effects of gravitational fields, which might affect the quantum mechanical phase of massive neutrinos (see for example [16–18], and also [19]).

in which $\delta A_p = A_p^{(1)} - A_p^{(2)}$. The corresponding oscillation length is therefore

$$L = \frac{2\pi}{\frac{\Delta m^2}{2p} + \delta A_p p} = \frac{L_M L_{LQG}}{L_M + L_{LQG}}. \quad (\text{III.4})$$

Here, $L_{LQG} = 2\pi/\delta A_p p$ is the oscillation length induced by loop quantum gravity corrections. It is worth noting that in Ref. [4] the oscillation length of neutrinos is $\sim p^{-2} l_{Pl}^{-1}$, a result obtained by imposing $\mathcal{L} \sim p^{-1}$ (see also the recent papers [20,21]). We shall take a different point of view in which the scale length \mathcal{L} is a free (and universal) parameter. The transition probability is

$$P_{\nu_e \rightarrow \nu_\mu} = \sin^2 2\vartheta \sin^2 \left(\frac{\pi \Delta r}{L} \right), \quad (\text{III.5})$$

being Δr the distance travelled by neutrinos and ϑ the vacuum mixing angle. Loop quantum gravity corrections are relevant when $L_M \approx L_{LQG}$, i.e.

$$\delta A_p \approx \frac{\Delta m^2}{2p^2}. \quad (\text{III.6})$$

Let us discuss separately the case $\kappa_2 = 0$ and $\kappa_1 = 0$.

- $\kappa_2 = 0$. Denoting with $\delta\kappa_1 = \kappa_1^{(1)} - \kappa_1^{(2)} \sim \mathcal{O}(1)$, and using Eq. (III.2), one gets

$$\mathcal{L} = \frac{2l_{Pl}\delta\kappa_1 p^2}{\Delta m^2}. \quad (\text{III.7})$$

For solar neutrinos, $\Delta m^2 \sim 10^{-10} \text{eV}^2$ and $p \sim \text{MeV}$, so that

$$\mathcal{L} \sim 10^{-6} \text{eV}^{-1}. \quad (\text{III.8})$$

In the case of atmospheric neutrinos, $\Delta m^2 \sim 10^{-3} \text{eV}^2$ and $p \sim \text{GeV}$, which implies

$$\mathcal{L} \sim 10^{-7} \text{eV}^{-1}. \quad (\text{III.9})$$

According to previous discussion, these values have to be discarded since no evidence of loop structure of space has been observed at this scale.

- $\kappa_1 = 0$. By defining $\delta\kappa_2 = \kappa_2^{(1)} - \kappa_2^{(2)} \sim \mathcal{O}(1)$, and using Eq. (III.2), one obtains

$$\mathcal{L} = \frac{\sqrt{2\delta\kappa_2} l_{Pl} p}{\sqrt{\Delta m^2}}. \quad (\text{III.10})$$

For solar neutrinos it follows

$$\mathcal{L} \sim 10^{-18} \text{eV}^{-1}. \quad (\text{III.11})$$

whereas for atmospheric neutrinos one gets

$$\mathcal{L} \sim 10^{-18} \text{eV}^{-1}. \quad (\text{III.12})$$

In both cases, the scale length \mathcal{L} belongs to the bound derived by Alfaro and Palma (I.6).

We conclude noting that loop quantum gravity provides us a scheme in which flavor oscillations can occur even for massless particles or for massive but degenerate neutrinos.

IV. CONCLUSION

Despite the concrete difficulties to probe quantum gravity effects occurring at the Planck scale, recently there has been an increase in the conviction that quantum gravity should predict a slight departure from the Lorentz invariance, which manifests itself in a deformation of the dispersion relations of photons and fermions. Such results have been indeed inferred in loop quantum gravity [3–5] and string theory [2]. In both approaches, a scale length, characterizing the scale on which new effects are non trivial, appears. The natural arena for probing the occurrence of the new scale is provided by gamma ray bursts physics, and by the observed GZK limit anomaly and related processes [7] (see also Refs. [22,23] for different contexts).

The aim of this paper has been to infer bounds on the length scale \mathcal{L} occurring in the AMU theory. Such bounds have been determined in the context of Cerenkov effect and neutrinos oscillations. Our analysis yields

$$\mathcal{L} \gtrsim 10^{-18} \text{eV}^{-1}.$$

Remarkably, it is compatible with bounds determined in Ref. [7]. Thus, corrections induced by loop structure of space becomes relevant for distances bigger than 10^{-18}eV^{-1} , an estimation that gives some hope to bring quantum gravity effects to the realm of experimental results².

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²To estimate the order of magnitude of \mathcal{L} is certainly of current interest, and, in fact, many attempts have been proposed in different systems. A stringent bound on $k_1 l_{Pl}/\mathcal{L}$ has been recently inferred by Sudarsky, Urrutia and Vucetich in Ref. [22]. Using their notation, we have

$$k_1 l_{Pl}/\mathcal{L} \rightarrow \Theta_1 l_{Pl} m$$

with $\Theta_1 < 10^{-5}$. Such a bound has been derived for the mobile scale, i.e. \mathcal{L} is fixed by the mass of particles $\mathcal{L} \sim 1/m$ (the nucleon mass in the case of Ref. [22]). Results of Ref. [22] seem to be very promising to probe quantum gravity effects as following from AMU theory.

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